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A praxeological model of reference related to costs calculation: comparison with the ones developed in a research and study path at university level

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Resumen	Este trabajo se encuadra dentro de una investigación más amplia cuyo objetivo general es enseñar matemática para no matemáticos en la universidad, adoptando las nociones centrales de la Teoría Antropológica de lo Didáctico (TAD). Se propone un modelo praxeológico de referencia sobre el cálculo de costos de un micro-emprendimiento y se lo compara con el modelo efectivamente desarrollado. La pregunta generatriz estudiada ¿Cómo calcular los costos en un micro-emprendimiento? involucra praxeologías matemáticas y económicas. Este modelo permite cubrir una parte del programa de estudios de un curso de Cálculo del nivel universitario. **Palabras clave**: cálculo, universidad, modelo praxeológico de referencia, teoría antropológica de lo didáctico.
Abstract	This work fits within a wider research whose general objective is teaching mathematics to non-mathematicians at the university, taking central assumptions of the Anthropological Theory of the Didactic (ATD). This work proposes a praxeological model of reference related to a micro-entrepreneurship costs calculation in comparison with the effectively developed model. The generative question How to calculate micro-entrepreneurship costs? links mathematical and economic praxeologies. This model allows to go through a part of the study programme of a university calculus course. **Keywords**: calculus, university, praxeological model of reference, anthropological theory of the didactic.
Resumo	Este trabalho enquadra-se dentro de uma investigação mais ampla cujo objetivo geral é ensinar matemática para não matemáticos na universidade, adotando as noções centrais da Teoría Antropológica do Didático (TAD). Propõe-se um modelo praxeológico de referência sobre o cálculo de custos de um micro-empreendimento e compará-lo com o modelo efetivamente desenvolvido. A pergunta geradora estudada: Como calcular os custos num micro-empreendimiento? envolve praxeologías matemáticas e económicas. Este modelo permite cobrir uma parte do programa de estudos de um curso de Cálculo do nível universitário. **Palabra-chave: cálculo, universidade, modelo praxeológico de referência, teoria antropológica do didático.**

1. Introduction

This work is part of a didactic research developed at university level. The institution is a public Argentine university, Universidad Nacional del Sur (UNS). Mathematics teaching characteristics at UNS satisfy the conditions that the ATD describes as visits to works. One of the objectives of the didactic research that is being realised refers to the analysis of the possibility conditions to deal with this phenomenon at least locally, taking the developments of the ATD such as the notions of research and study paths (RSP) and praxeological model of reference (PMR).

The RSP proposed has been implemented in two opportunities (N=38 and N=35 students) in a mathematics course corresponding to Enterprise Administration and Public Accountancy at the UNS. It begins with a question related to economy: *how to calculate micro-entrepreneurship costs?*

Firstly, a description of the PMR is presented in this article. It has been designed previous to these implementations. It would be possible through this RSP not only to study different institutional culture works, but also to produce the emergence of other researches. This model also helps us to detect which these "works" are and which of them are linked to the proposed curriculum.

In this PMR the mathematical and economic organizations giving answer to the generative and derived questions are presented. Particularly, the RSP would allow rebuilding mathematical organizations related to two-variable Calculus and an important part of the institutional proposed curriculum. Also, it would be possible to manage the involved organizations integration.

Secondly a general description of the developed RSP is commentated. Finally it is possible to make a comparison between the reference model and the implemented one giving answer to these questions: which paths do the teacher and the students go through? does this path coincide with any presented in the PMR? does it lead really to the study of part of the proposed curriculum? which are the effectively studied praxeologies?

Thus it is confirmed that the path followed by the groups of study is one of the possible paths detected by the PMR, between other conclusions.

In this way we highlight the importance of the PMR as an instrument of didactic and praxeological analysis even if the realization or not of an RSP.

2. Previous Researches

Different Calculus teaching researches are related to the didactical phenomenon of mathematical content disconnection, rigidity and atomization of mathematical organizations (Lucas, 2015; Trigueros, 2005). At university level and from ATD point of view, some investigations propose an inquiry-based calculus teaching (Barquero, 2009; Serrano, Bosch & Gascón, 2007).

Lucas (2015) points up the main characteristics of an MER (epistemological model of reference) structured by an activity diagram containing different mathematics paths in an RSP. The importance of the MER is emphasised as a

provisional and relative system of reference from which the formulation of new didactic problems is possible.

Costa, Arlego and Otero (2013) design and implement a teaching by RSP in the context of an Engineering Faculty in Argentina. Recovering the vectorial calculus sense and *raisons d'être* was attempted in this path, integrating physic notions. The path permitted the approach to mathematical organizations included in the institutional curriculum.

On the other hand, Otero et al. (2013) promote and analyse the possibility conditions of a teaching in the sense proposed by the ATD at university level.

The PMR proposed in this work shows different paths in a teaching by RSP, in a regular math course at university level, integrating two-variable Calculus and Economy notions. Some results obtained in the RSP implemented can be found in Salgado, Otero and Parra (2017).

3. Theoretical framework

The reference knowledge analysis involved in a Research and Study Path (RSP) is an action the didactic researcher must be able to realize before carry out the teaching by RSP. This knowledge integrates the praxeological model of reference (PMR) (Chevallard, 2013) consisting in an always provisory and open analysis of the organizations or praxeologies of one or more disciplines, mathematics and economy in this case, the researcher would meet, or meet again, studying a question Q_0. What he must know or give himself the liberty to learn about Q_0 is not identified with what a professor knows or with the way he would answer Q_0. On the contrary, the researcher must adopt a precognitive posture, "ask the world" in which he is situated. In summary, the PMR underlies the whole teacher, student and researcher activity.

A PMR elaboration importance lies in its utility as a tool of didactic and praxeological analysis, being always of dynamic nature. Following up on the praxeological analysis implies formulating didactic questions as: Where does this "work" come from? Why is it there? How has it been learned by the institution? Which transformations has it suffered? On the other hand, all didactic analysis implies considering how is the praxeology pretended to teach so that it leads to identify the structure and the studied work functioning (Lucas, 2015).

A PMR elaboration is also important facing the didactic phenomenon that Chevallard (2005) metaphorically describes as *monumentalism*. Monumentalism is the predominant epistemological view of the traditional mathematics and other disciplines teaching, consisting in showing the knowledge already constructed. It is characterised by considering knowledge as a monument, that only could be admired, venerated, but never altered or deconstructed. Thus there is only one and monolithic mathematics and it has been and will be always the same. This monumentalistic epistemological conception promotes the idea that the mathematical knowledge is

not the problem when teachers are organizing teaching and also, that mathematics is evident and transparent in the teaching process. Contrary, the ATD advocates for a new, emergent and non-monumental paradigm where the research and questioning the world are the rule. In this paradigm, knowledge is the product of questions related to social necessities into a given historical moment, affected by conditions and constraints of different levels, included humanity, society, pedagogy, social institutions, etc. This includes the moment where some mathematical knowledge has to be taught and studied at university in a certain moment. One relevant consequence originated by this epistemological point of point is that mathematics is useful treating everyday problems, like decisions making in economical, commercial, domains.

4. Description of the reference institution and the course

The institution is a public Argentine university, Universidad Nacional del Sur (UNS), with a departmental organization, so that exclusively the Mathematics Department lecture mathematics courses of all majors. Also, the courses are developed in a four-month form, the first one from March to June and the second one, from august to November. Particularly the mathematics courses have a theory-practice modality, where a professor lectures the theory class and one or more assistants are in charge of the practice class. This institution is characterised by a monumental viewpoint of mathematics teaching, like all those who respond to an official curriculum.

The proposed RSP pretends to go through the contents of the first year subject Matemática IIC (MIIC), second four-month period, in the degree of Enterprise Administration and Public Accountancy at the UNS. This subject study programme is divided in four modules:

- Sequences and series
- Linear equations systems. Matrix Algebra
- Several-variable functions
- Extremes of several-variable functions. Linear Programming.

The main objective of MIIC is to acquire notions related to the more-than-one variable functions analysis, placing emphasis in its application to administration and economic concrete problems. The programme is developed in two theoretical and two practical weekly classes of two hours each. The evaluation modality includes the approval of partial exams and a final one, both written ones.

5. Praxeological model of reference related to costs calculation

5.1. Generative question analysis

The PMR described in this work allows us to delimit and analyse the possible research and study paths that arise from the question Q_0: *How to calculate micro-*

entrepreneurship costs? We will see that the elaboration of a possible answer to Q_0 results in the study of different mathematical and economic organizations, for example, a mathematical organization (MO) related to differential Calculus and other economic one (EO) linked to costs calculation.

The starting question Q_0 allows the emergence of multiple economic questions as, for example, what is a micro-entrepreneurship about? which are the generated costs? which is the purpose of doing a costs calculation?, among others.

First questioning belongs to the notion of micro-entrepreneurship. If its definition and central characteristics are looked up in the Internet, one can find statements such as: micro-entrepreneurship is an earning generating company owned and run by its own entrepreneurs. They themselves work at these companies, in general, without employees. It is an individual or family project requiring very low capital investment. The notion of micro-entrepreneurship is associated to the idea of micro-credits, born in Bangladesh in the 70's when the economist Muhammad Yunus gave economic aid to a group of poor women to carry out a small business. This initiative led to the foundation of a social bank orientated to the poorest so they could go out of their misery.

The second questioning comes from the notion "costs", understood as all the money needed to produce and commercialise products, it is all that will be used to manufacture the products that will be sold later. So that, not everything that is an expense represents a cost. An important part corresponds to the necessary capital in order that the micro-enterprise could work. The capital is the value of what is possessed, whereas the cost is the value of what the company uses to generate a product.

Finally, the questioning about "why" a costs calculation must be done is very important for the micro-entrepreneur. In general, a costs study and calculation are needed to fix sale prices. Most enterprises fix their prices, principally and exclusively, taking into account costs, but also costs determination is elementary for decision making because it allows, for example:

- Determine business results (profit or loss)
- Evaluate the level of competitiveness
- Determine the marginal contribution and the equilibrium point
- Plan future investments
- Analyse the incorporation of new products.

Only over costs it is possible to make an enterprise control or manage because the sale price of a product can be fixed by the market.

5.2. First possible answers to the generative question

The first stage in the construction of an answer to Q_0 refers to the different hypotheses analysis that will determine, for example, the variables of the system. De Renolfi and Cardona (2007) affirm that different alternatives require the application of

different types of costs, it does not exist an only one cost but different ones for different purposes. These authors consider that different hypotheses related to costs calculation exist; namely,

- H_1: costs behaviour following an independent variable
- H_2: costs relation to the possible product allocation
- H_3: calculation extension
- H_4: costs considering the relationship to the moment of the calculation
- H_5: the relationship to the decision making.

Taking into account the above-mentioned hypotheses, different costs considering each one are described below:

H_1: *costs behaviour following an independent variable*. This hypothesis is considered when the system to model refers to an enterprise whose objective is to fix the article sale price, to cover the minimal cost. Under H_1, the cost depends on one or more independent variables, for instance, production, level of activity, supplies, etc. In this case, costs are classified as fixed (FC) and variable (VC) costs. As a consequence, the total cost is the addition of both: C=FC+VC, which represents a first answer to Q_0. A fixed cost is that whose amount is constant, whatever the value of the independent variable. This does not mean that it is invariable in the long-term; for example, rent, insurances, etc. A variable cost is the one modified in relation to the independent variable value; for example, raw material, direct manpower, etc.

H_2: *Considering costs relation to the possible product allocation*, costs are classified as direct and indirect. Direct ones are those identified in every produced article, either in its physical aspect, or in its value. They are produced when the activity is carried out; they depend on the realization or not of it, for example, supplies, manpower related to the activity, etc. Indirect costs are those related indirectly to the articles. They are produced independently of the realization or not of a certain activity; for example, taxes, general accounts, fuel expenses, etc. In this case, the cost (C) is given by the addition of the direct (DC) and indirect (IC) costs: C=DC+IC, which represents a second answer to Q_0.

H_3: *considering the calculation extension*, costs are classified as total and partial. Total ones are those involved in the totality of a certain activity, while partial ones are those referring to a specific aspect of the activity. In this case, the cost (C) is given by the addition of the total (TC) and partial (PC) costs: C=TC+PC, which represents a third answer to Q_0.

H_4: *considering the relationship to the moment of the calculation*, costs are classified as real and estimated. Real costs also called historical, retrospective or resulting are those in which the enterprise incurred in a past activity. They are used to evaluate past actions and to control the management of the enterprise. Estimated costs, also called future, prospective or budgeted are those that could happen in a future situation during the manufacturing of a product.

H_5: *considering the relation to the decision making*, costs are classified as marginal, incremental, relevant and opportunity costs. Marginal cost is the cost of the last produced article or the cost required to increase the production in a unit. Incremental cost refers to how much the cost was raised on having increased the activity at a certain level. Relevant costs are those that have a special opportunity for every decision making. Finally, opportunity costs refer to the value of the rent that might be obtained if the resource was used in its better alternative.

The unitary cost is not included in this classification made by De Renolfi and Cardona. It represents the production cost of an article and it is important to fix the sale price. However, searching the Internet how to calculate the costs of a micro-entrepreneurship, principally the fixed, variable, total and unitary costs, are mentioned.

5.3. Construction of two models

After performing a costs analysis, the micro-entrepreneur estimates total production costs; he puts up all data in tables -considering hypotheses H_1 to H_5, or even a combination of them- and performs simple arithmetic operations to answer Q_0, by adding the registered information. This leads to consider both a mathematical organization related to arithmetic operations and an economic one referred to costs according each hypothesis.

Supposing that one wants to calculate the total costs, under H_1, it is necessary to determine the fixed and variable costs for the manufacturing of a product. Whereas the fixed ones (rent, salaries, electricity, gas, etc.) can be calculated monthly, the variable ones (materials for the production, packaging, delivery costs, etc.) can be calculated by produced unit. Table 1 shows a list of costs if only one product is manufactured:

Fixed	per month	Variable	per unit
Gas	F1	Materials	V1
Electricity	F2	Labels	V2
Rent	F3	Package	V3
Taxes	F4		
Fixed Costs per month	FC=F1+F2+F3	Total Variable Costs	VC=V1+V2+V3

Table 1. Costs in the manufacturing of an article

The total fixed monthly costs FC emerges from the addition of all fixed costs. The total variable costs per unit VC is the addition of the different manufacturing expenses. If x is the number of manufactured products in a month, the total cost is given by: $C = FC + VC \cdot x$, with FC, VC no negative real numbers.

If on the other hand, two articles are manufactured, variable costs are calculated per unit (see Table 2). The fixed monthly costs FC emerge from the

addition of all fixed costs. VC1 and VC2 are the variable costs to manufacture articles 1 and 2, respectively.

Fixed	Per month	Variable		Per unit	total per unit
Gas	F1	Art. 1	Materials	V11	VC1=V11+V12+V13
Electricity	F2		Labels	V12	
Rent	F3		Package	V13	
Taxes	F4				
		Art. 2	Materials	V21	VC2=V21+V22+V23
			Labels	V22	
			Package	V23	
Fixed Costs	FC=F1+F2+F3+F4				

Table 2: Costs in the manufacture of two articles

If x and y are the number of articles 1 and 2 manufactured in a month, respectively, the total cost is given by: $C = FC + VC1 \cdot x + VC2 \cdot y$, with FC, $VC1$, $VC2$ no negative real numbers.

5.4. Derived questions from Q_0

The search of an answer to Q_0 originates more questions, such as:

Q_1: How many articles can one make with a certain amount of money?

Q_2: Which is the marginal cost?

Q_3: How does the total cost change considering a modification in the variable costs?

Q_4: Which are the maximum and the minimum cost?

A questioning about how to answer Q_1 to Q_4 emerges, what techniques to use, that we resume in Q_5: How do we reply to each of Q_1 to Q_4?

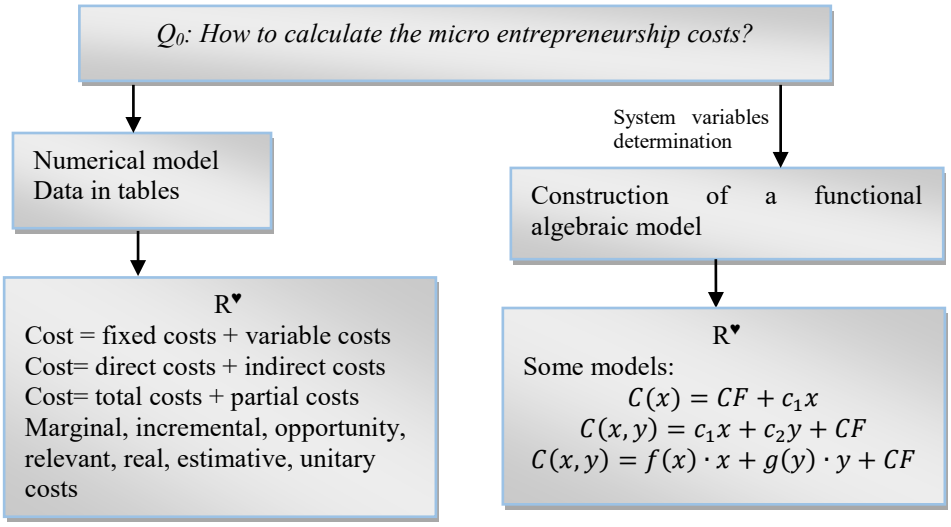
Considering a numerical model, where the micro-entrepreneur performs a regular register of the costs, one can deduce conclusions from these data in tables; such as, estimating short term costs, calculating marginal costs, analysing cost variation, determining how many articles can one manufacture with a certain budget, etc. From specific data, estimations or exact calculation will possibly be done in order to answer Q_1 to Q_4, among other questions.

Considering a functional algebraic model, if the micro-entrepreneurship, for example, is engaged in the production of two articles and it must face fixed and variable costs, the model $C = FC + VC1 \cdot x + VC2 \cdot y$, with FC, $VC1$, $VC2$ no negative real numbers, can be written as:

$$C(x,y) = c_1x + c_2y + FC$$

with FC representing fixed costs (rent, electricity, etc.), c_1, c_2 variable ones, and $C(x,y)$ the cost function with variables x and y. It will be showed that this model also allows us to answer Q_1 to Q_4.

Up to here, two possible paths are detected (see Scheme 1). They allow to find answers not only to the generative question, but also to the derived ones.



Scheme 1: Two possible paths detected

Considering the functional algebraic model, one can describe an answer to Q_1. Supposing a certain amount of money or budget for expenses, the answer will indicate which the production with this budget is, which possibilities with this constraint are contemplated.

If one has an amount of *K* monetary units to carry out the project and to use it completely, one models this situation by the equation $C(x,y) = K$, which represents a level curve of the surface whose equation is $z = C(x,y)$. The curve corresponds to a set of points in the *XY* plane for which $z = K$, with K a positive constant. From an economic point of view, this is an isocost curve (Mochón and Beker, 2003) or boundary of possibilities. Every point (x,y) belonging to it brings a production level for which the cost is constant. If a point (x,y) doesn't belong to the curve, it means that, either the budget is exceeded or it is not used totally. Thus, all the points verifying $C(x,y) = c_1x + c_2y + FC = K$ represent possible solutions to Q_1. In this way, this analysis that answer Q_1 allows to get into an economic organization related to costs and a mathematical one referred to two-variable calculus and notions of analytic geometry in the plane.

With regards to Q_2*: which is the marginal cost?* this means: how much the cost varies given an increase in one unit in the production level? In the case of the functional algebraic model in two variables considered here, the question can be reformulated as: *Given a fixed number of manufactured articles of one type, how*

does the cost change if an increase in the number of manufactured units of the other type happens?

The rate of change of a variable in relation to another, here, how does C change given an increment of x or of y, requires to get into the "work" of derivative. In this way, given the function $z = C(x, y)$, one can answer Q_2 by taking partial derivatives: $\frac{\partial C}{\partial x}$ and $\frac{\partial C}{\partial y}$. These derivatives evaluated in a point (x_0, y_0), have an economic interpretation, represent the approximated change of C for each unit increase in x (or y) keeping y (or x) fixed, which are called marginal costs.

For example, $\frac{\partial C}{\partial x}(x_0, y_0)$ represents the approximated change of C for each unit increase in x, keeping y fixed, at the moment in which the production level is $x = x_0$ and $y = y_0$.

Similarly, $\frac{\partial C}{\partial y}(x_0, y_0)$ represents the approximated change of C for each unit increase in y, keeping x fixed, at the moment in which the production level is $x = x_0$ and $y = y_0$.

The question Q_3*: How does the total cost change considering a modification in the variable costs?* states that the parameters c_1 and c_2 can change. Supposing that one of these parameters changes and the other remains constant, the question can be reformulated as: *considering a fixed manufacturing cost of one of the articles, how does the total cost vary if an increase or decrease in the manufacturing cost of the other article happens?*

If c_1 and c_2 depend on the production, for example, $c_1 = f(x)$ and $c_2 = g(y)$, $C(x, y) = f(x) \cdot x + g(y) \cdot y + FC$, then the approximated changes of C are given by:

$\frac{\partial C}{\partial x} = f'(x) \cdot x + f(x) = c_1'(x) \cdot x + c_1$ and $\frac{\partial C}{\partial y} = g'(y) \cdot y + g(y) = c_2'(y) \cdot y + c_2$.

In this way, answering Q_2 and Q_3, one gets into an economic organization related to costs, specifically marginal costs, and a mathematical one referred to two-variable differential calculus.

Finally, considering the functional algebraic model, an answer to Q_4*: Which are the maximum and the minimum cost?* can be found studying a mathematical organization referred to two-variable Differential Calculus.

The search of an answer to Q_4 can generate more questions: how to calculate maximum (minimum) cost? what techniques to employ?, what constraints do the independent variables x and y have, if any? These questions are solved using techniques to find relative or absolute extreme values of the cost function $C(x, y)$. If there are not constraints for the variables, one can use techniques to find local or relative extreme values of $C(x, y)$, named stationary values. One of these

techniques, under some assumptions, refers to finding the points where the partial derivatives are simultaneously equal to zero:

$$\begin{cases} \dfrac{\partial C}{\partial x} = 0 \\ \dfrac{\partial C}{\partial y} = 0 \end{cases}$$

On the other hand, if there are constrains, one employ techniques of constrained optimization. In the case that we are considering, the variables take positive values, then one has at least the restrictions: $x \geq 0$, $y \geq 0$.

Another possible constraint is that of fixing the production in a units so that one has to find the extremes of the cost function knowing that the total production is a units. This leads to search for the extremes of $C(x, y)$ subject to the equality $x + y = a$, with a positive integer, or the constraint could be an inequality $x + y \leq a$, if the production is up to a units. One can use techniques of extreme values calculation to solve this kind of problems, for example, the method of Lagrange multiplier -constraints given by one or more equalities- or techniques of linear programming -linear constraints, given by inequalities and linear cost function-, by solving the following systems respectively:

Extreme values with constraints given by equalities

$$\begin{cases} C(x, y)\text{: function to optimise} \\ \text{constraint given by: } a_1 x + a_2 y = a \\ \text{with } x \geq 0, y \geq 0 \end{cases}$$

Linear programming

$$\begin{cases} C(x, y)\text{: linear function to optimise} \\ \text{constraints of the form: } a_1 x + a_2 y \leq a \\ \text{with } x \geq 0, y \geq 0 \end{cases}$$

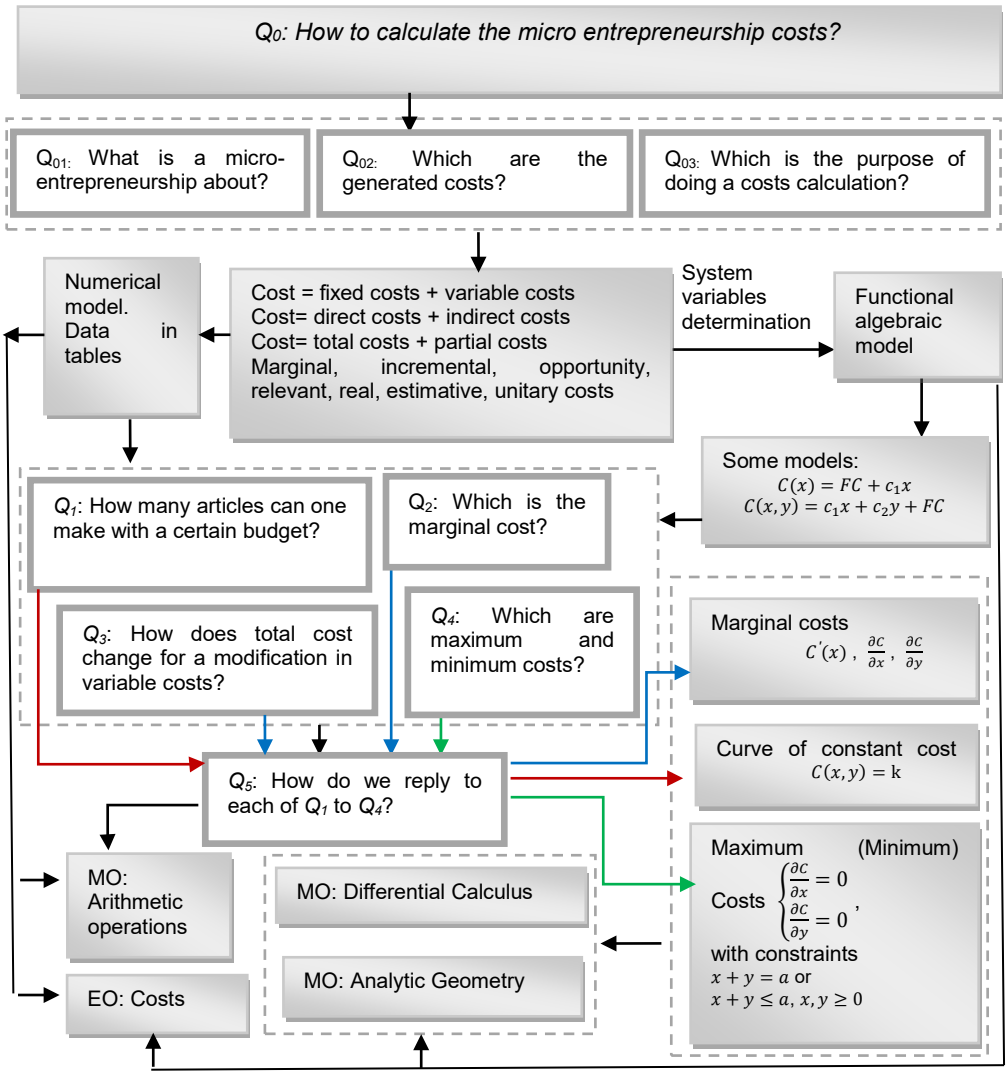


Figure 1: Generative and derived questions. Possible MO and EO to rebuild in the SRP

It is easily seen that considering the functional algebraic model the search of answers to questions Q_2 to Q_5, leads to the study of mathematical organizations referred to two-variable Differential Calculus, which includes dealing with functions of two variables, the calculation of partial derivatives and extreme values. The generative and derived questions are shown in Figure 1, as well as the possible organizations to rebuild in the research and study path.

6. Implemented Research and Study Path

The research and study path proposed was implemented in two opportunities, implementation 1 (IMPL1) and implementation 2 (IMPL2) during 2014 and 2015 respectively, in a course of Matemática IIA (MIIA) in the degree of Enterprise Administration and Public Accountancy at the UNS. Actually some changes have been carried out in the study programmes of these majors so that MIIA has been modified and was renamed as MIIC.

A detailed description of each performed path is not included in this work whose only objective is to compare these paths with the praxeological model of reference described in the last section, indicating principally which trajectories have been taken in IMPL1 and IMPL2 respectively.

Students were distributed in groups of 3 or 4 members each. Beginning the research and study path with the generative question Q_0, an important number of derived questions arose (see Table 3).

Derived questions in IMPL1	**Derived questions in IMPL2**
Q_1: Which is the cost to manufacture every article? Q_2: How can we predict the manufacturing cost of a certain amount of articles of each type? Q_3: How many articles of each type could we manufacture with a certain amount of money? Q_4: Which is the cost to produce one more unit of each product? $Q_{4,1}$: How is the relation between marginal and variable costs? $Q_{4,2}$: If total cost function were quadratic, how we could calculate marginal costs? $Q_{4,3}$: What are partial derivatives? How to calculate them? $Q_{4,3,1}$: Is the increase of a function equal to the derivative? Is it correct to write $C'(x_0, y_0) = C(x_0 + 1, y_0) - C(x_0, y_0)$? $Q_{4,3,2}$: If a partial derivative represents the slope of a tangent line, which is this tangent line? $Q_{4,3,3}$: Which are the two variable-function, tangent line and tangent plane graphics? $Q_{4,4}$: If a function is derivable in a point, this means that it is continuous in this point? $Q_{4,4,1}$: Derivable is the same that differentiable? Q_5: How does the demanded quantity vary with respect to the price? Q_6: How to minimize the costs, to produce more quantity at the same cost?	Q_{01}: What is a micro-entrepreneurship about? Q_{02}: Which are the generated costs? Which types of costs exist? Q_{03}: How to calculate them? Q_1: How to calculate the marginal cost? $Q_{1,1}$: Which would be the cost function if two articles are manufactured? $Q_{1,1,1}$: How is the proposed cost function graphic? $Q_{1,1,2}$: Which is the domain? $Q_{1,2}$: How to calculate the marginal cost of a two-variable function? $Q_{1,2,1}$: How to calculate partial derivatives? $Q_{1,2,2}$: If a function is derivable, it is also continuous? $Q_{1,2,3}$: What is a directional limit and what situations are presented in the cost calculation? $Q_{1,2,4}$: We know that the tangent line is related to the covered topics, how could we apply it to the micro-entrepreneurship example? Q_2: How many articles can we manufacture with a certain budget? Q_3: How does the time influence on our costs? Q_4: How does the cost vary in case of an increment in the number of manufactured articles? $Q_{4,1}$: What is a differentiable function? $Q_{4,2}$: How can I demonstrate that a function is differentiable? Q_5: How many articles do I have to sell monthly in order to have no loss? What is the minimum to produce in order to have no loss? $Q_{5,1}$: How to find the extremes of a two-variable function? $Q_{5,2}$: Which is the maximum utility that a micro-entrepreneurship can obtain? Which is the number of articles of every type that leads to that utility? $Q_{5,3}$: Which is the maximum or minimum utility according to a given budget? Idem for the maximum or minimum production. Q_6: How to calculate the total cost based on the marginal cost?

Table 3: Derived questions from Q_0

Referring to the trajectory gone through by the study groups (teacher and students) and the studied mathematical and economic organizations, both experiences showed similar results. The study group at each opportunity opted to a

functional algebraic model as an instrument to answer Q_0 and the derived questions mentioned in Table 3. They determined the variables of the system and formulated a cost function that would allow to calculate the costs in a specific micro-entrepreneurship, performing simple arithmetic calculation to estimate some parameters (fixed and variable costs). Students choose this model because they expressed being part of a mathematics course so that they had to formulate a expression as a function.

The answers to questions, using the functional algebraic model led to the study of a mathematical organization related to two-variable differential Calculus including different works of the institutional culture (see Table 4). In this way it was possible to cover part of MIIA study programme.

Studied works in IMPL1	**Studied works in IMPL2**
Two variable-functions. Partial derivatives. Differentiability. Maximum and minimum calculation with and without constraints. Costs: fixed, variable and marginal costs.	Two variable-functions. Partial derivatives. Differentiability. Directional derivative. Maximum and minimum calculation with and without constraints. Costs: fixed, variable and marginal costs.

Table 4: Studied works

Also, it was possible to rejoin an economic organization related to costs. Both study groups studied marginal costs, isocost curves and extreme values of the cost function always taking the functional algebraic model as a reference. Although they began with a question referred to a micro-entrepreneurship, they did not make a deep economic study and they focused on the mathematical details trying to put results into context.

7. Discussion

Firstly, the praxeological model of reference shows that Q_0 is open and generates multiple questions that can result in the study and research of mathematical and economic organizations. Two possible paths are detected in order to search an answer to Q_0 one path using a numerical model and another, a functional algebraic one. In the first one, adequate costs are considered according to the micro-entrepreneur objectives which lead to meet an economic organization related to costs and a mathematical one referred to arithmetical operations. In the second one, the variable number determination will imply the one or more variable functions study. Therefore, to answer the derived questions, for instance, one will get into the mathematical organization study related to two-variable differential calculus. Another possible path is to consider firstly the numerical model and then to decide to take the functional model. While on the other hand the path corresponding to a functional algebraic model was the only one followed by IMPL1 and IMPL2.

In the mathematics university level course where the research and study path is performed, the search of an answer to Q_0 and its derived questions would allow, at least, rejoining a mathematical organization relative to two-variable calculus and an economic one referred to costs. This fact was confirmed by the performed IMPL1 and IMPL2, leading to the study of these contents related to the two-variable differential calculus: partial derivatives of, differentiability, directional derivative and maximum and minimum calculation with and without constraints. It led also to the study of different types of costs such as fixed, variable and marginal costs.

The answers to questions Q_0 to Q_5, described in the praxeological model of reference, using the functional algebraic model, could lead to the study of a mathematical organization related to two-variable differential calculus, which are part of modules 3 and 4 of MIIC study programme. Referring to this point, the performed implementations confirmed us that a research and study path beginning with Q_0 is an instrument to cover part of this subject study programme.

Referring specifically on the questions, an important number of them arose from the generative one during both experiences, remarking the open character of Q_0. Furthermore, different questions arose, different than those presented in the praxeological model of reference, which led to cover the same contents.

At last, we remark that it would be possible to go over the institutional proposed curriculum using a different pedagogy.

7. Conclusions

In this work we have initially presented a praxeological model of reference belonging to a research and study path related to a micro-entrepreneurship costs calculation and then some details of the one corresponding to an implemented research and study path. Later we emphasise the most important characteristics shared by them.

Possible paths are detected and pointed out in the praxeological model of reference; namely the one using a numerical model and a functional algebraic one. Besides, it is shown that these paths would allow going through a part of the reference institutional proposed curriculum.

This praxeological model of reference also shows that Q_0 is open and generates multiple questions that can result in the study and research of mathematical and economic organizations.

Two possible paths are observed in order to search and find an answer to Q_0, one path using a numerical model and another, a functional algebraic one. We notice that the last one was chosen by the study group in both implementations.

In the mathematics university level course where the research and study path is performed, Q_0 effectively allowed rejoining a mathematical organization relative to

two-variable calculus and an economic one referred to costs. Therefore the answers to Q_0 and the derived questions, using the functional algebraic model, led to the study of mathematical organizations related to two-variable differential calculus, which are part of MIIC study programme. In this way, it was possible to go over the institutional proposed curriculum using a different pedagogy that introduce the mathematics in a university course where this discipline is not always well received by its students. Besides we make an alternative contribution to different proposed didactical researches in calculus teaching field (Sánchez-Matamoros, García & Llinares, 2008).

Further than the development of the research and study path, we remark the relevance of the praxeological model of reference as the didactic equipment of any teacher.

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